

# CONTROLLING THE DYNAMICAL SYSTEM OF MACHINE TOOL BY ELASTIC-DEFORMABLE SHAFTS MACHINING

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*In article has been presented machining process mathematical model of elastic-deformable shafts. It has been presented problem of optimal regulation of technological system in automated production systems too.*

## 1. INTRODUCTION

The significant group among different rotational parts establish parts with small stiffness which are made of alloy steel with high strength: shafts, axes, toothed shafts, torsional and elastic shafts, shafts-impellers of electric micro machines, precise and special tools (drills, reamers, screw-taps, boring bars) and other. Traditional application by turning of rests and other devices which increase stiffness doesn't enable sufficient dynamical strains decreasing and complicate a machining process, limiting their application in automated production.

There are lots of efficient ways of increasing the precision of parts with small stiffness machining and one of them is controlling precision taking into consideration their elastic-deformable state [1, 2]. According to considered [1] classification, following regulating factors (or their combination) are applied as regulating reactions: axial tension or tension with axis displacement, accessory force reactions, orientated toward compensation of machining process force factors; bending moments on abutments and bending-torsional force reactions.

During designing systems of automated control (SAC), which are intended for decreasing the reliability of gears' and machining precision, it is necessary to take into consideration the dynamical properties of control object (CO), which are determined first of all by all processes connected with elastic strains of technological system's (TS) elements. Much of a priori information about processes which take place in machines' dynamical systems, enables the analytical solution of identification task, and after that an experimental precisely definition of model's parameters.

## 2. SIMPLYFYING OF OUTPUT MATHEMATICAL MODEL OF DYNAMICAL TECHNOLOGICAL PROCESS OF MACHINING SYSTEM

One of the most urgent problems which appear during precision estimation and SAC designing is to work out the mathematical description of TS which is considered as CO. Dynamical system (DS) of turning is a TS – that means, a machine tool with technological process (TP) of turning, which is realized in it. In engineering practice of SAC calculation, representing the properties of dynamical elements and systems as typical dynamical links and their combinations is widely applied. In [2,3] they obtained relations for operator transmittance (OT) and mathematical models (MM) of DS in such form:

$$G_{Fg_y}(s) = K_{Fg_y} \frac{1 + B_0 G_\tau(s)}{1 + B G_\tau(s)}, \quad (1)$$

where:

$$K_{Fg_y} = \frac{K_{yF}}{1 + n_y h_{yy} + n_x h_{xy} + n_z h_{zz} k_{bz}}; B_0 = m_x h_{xx}; \quad (2)$$

$$B = \frac{m_x h_{xx} + K_{K_r} (m_y h_{yy} + m_x h_{xy} + m_z h_{zz} k_{bz})}{1 + n_y h_{yy} + n_x h_{xy} + n_z h_{zz} k_{bz}}; \quad (3)$$

$$G_\tau(s) = 1 - e^{-s\tau},$$

where  $\tau = 1/n_{wr}$  – delay time,  $n_{wr}$  – rotational speed of fixed headstock, and the factor of proportionality  $K_{yF}$  is defined by such relation [2]:

$$K_{yF} = \left( \frac{\partial g_y}{\partial F_{x1}} \right)_0 = - \frac{g_{y0} L_p^2}{4\pi^2 EI + F_{x1_0} L_p^2}. \quad (4)$$

In case of lengthwise grinding, a grinding wheel is considered as equivalent edge and its approach angle is  $\kappa_r = 90^\circ$ . For process of lengthwise turning with edges  $\kappa_r = 90^\circ$  and lengthwise grinding, the relation (3) simplifies:

$$B = \frac{m_x h_{xx}}{1 + n_y h_{yy} + n_x h_{xy} + n_z h_{zz} k_{bz}}.$$

OT of the object by application of other force control reactions is reduced to analogical form and it differs from (1) only with factor's of proportionality expressions, which data is included in [2]. Introducing a generalized factor of proportionality  $K_{0Fg_i}$ , OT of the object by different additional force reactions can be defined in following form:

$$G_{Fg_i}(s) = K_{0Fg_i} \frac{1 + B_0 G_\tau(s)}{1 + B G_\tau(s)}. \quad (5)$$

A control defect by applying the tensile force (what is effective only for defined parts type dimensions) is a small value of factor of proportionality according to controlling reaction that results in a small susceptibility of SAC to that reaction. The efficiency of controlling and sensitivity of SAC increase markedly by applying other controlling force reactions, e. g. additional bending forces and bending moments. There is also another way of additional reactions to elastic-deformable state: applying electromagnetic forces which appear as the result of electromagnetic field and passing current through that part.

The proposed regulating reaction is taken into consideration in a vector of additional force reactions as components of  $\Delta F_{em}(s)$  – an additional force which is oriented on compensation of TS elastic strains along Y axis. OT of the model for that reaction is reduced to OT form (5). Besides, the object's factor of proportionality is defined with such form:

$$K_{F0g_y} = \frac{h_{yy}}{1 + n_y h_{yy} + n_x h_{xy} + n_z h_{zz} k_{bz}}. \quad (6)$$

Comparing OT relations according to control reaction in the shape of feed with object's OT with other control force reactions, it can be seen, that in OT according to (5) in contradistinction to OT for feed channel:

$$G_{v_f g_i}(s) = \frac{\Delta g_i(s)}{\Delta v_f(s)} = \frac{K_{1v_f g_i} G_\tau(s)}{s[1 + B G_\tau(s)]},$$

multiplier  $\frac{1}{s} \cdot G_\tau(s)$  doesn't exist [2, 3]. Hence, the model's inertia according to channel of additional force reactions turn out to be lower. It can be confirmed basing on analysis of frequency characteristics of base model. Applying Euler transformation to exponential function and taking  $K_{0Fg_y} = 1$ , object's frequency OT is obtained:

$$G_0(j\omega) = \frac{(1 + B_0 - B_0 \cos \omega\tau) + jB_0 \sin \omega\tau}{(1 + B - B \cos \omega\tau) + jB \sin \omega\tau},$$

According to it, they defined relations for amplitude frequency characteristic (AFC) and phase frequency characteristic (PFC) of the object:

$$A(\omega) = \frac{\sqrt{(1 + B_0 - B_0 \cos \omega\tau)^2 + (B_0 \sin \omega\tau)^2}}{\sqrt{(1 + B - B \cos \omega\tau)^2 + (B \sin \omega\tau)^2}};$$

$$\varphi(\omega) = \operatorname{arctg} \frac{B_0 \sin \omega\tau}{1 + B_0 - B_0 \cos \omega\tau} - \operatorname{arctg} \frac{B \sin \omega\tau}{1 + B - B \cos \omega\tau}.$$

It is obvious, that dynamical properties of the object depend on factors  $B$  and  $B_0$  ratio.

If  $B \cong B_0$ , model's properties for taken assumptions are close to characteristics of proportional link. In general case, AFC and PFC are periodical functions with period of  $\tau$ -multiplication. Calculations of model's frequency characteristics according to mentioned relations show, that logarithmic amplitude characteristic (LAC) of base model differs from LAC of proportional element in relatively small limits. For example, with  $B=0,6$  and  $B_0=0,2$  the maximum deviation doesn't exceed  $3,9$  dB; in case, that  $B_0=1$ , deviations grow with simultaneous coefficient  $B$  increasing, when  $B=0,5$ , the maximum deviation doesn't exceed  $6,1$  dB.

### 3. EXPERIMENTAL RESEARCH ON DYNAMICAL CHARACTERISTICS

The aim of the experimental research on TS dynamical characteristics during elastic-deformable parts' turning consist in estimation of adequacy the object's MM (obtained basing on analytical methods of identification) to original.

The elastic-deformable part with small stiffness was applied the static forces, which simulate the cutting force: axial and with axle displacement by tensile forces, bending moments, additional forces with elastic dampers counteraction on the special stand. Elastic strains of TS elements were determined with micrometric handwheels. On Fig. 1 they presented the block diagram of experimental stand, where transient processes in DS (including part with small stiffness during machining with tension) are studied.

The stand is organized with use of lathe II 616P and include the part with small stiffness 1, fixed in holder 2 and sleeve clip 3 of tailstock 3, which include the tensile mechanism in shape of air-powered force cylinder 5. Cavities of the air-powered cylinder are connected to feeding line by electro pneumatic valves 6 and block 7 to air preparing. The part is machined by the edge 8, which is fixed in elastic tool holder 9 (dynamometer) on the machine's slide 10. On the bed of the machine tool there is an additional device (settled with use of bracket 11) which defines strains resulted from parts' vibrations. The device's bolt 12 is settled in ball slides of sleeve 13, the spring 14 generates tension.

They applied the measurement-recording complex which include two-component dynamometer 9, tezo-station 7 (type 8ANCz-7N), indicators 16 and 17, oscillograph 18 to transient characteristics of components  $F_c$  and  $F_p$  change recording.

The measuring complex feeding proceeded from stabilizers 19; vibration strains were measured by apparatus WI 6-6TN, which consists of : power voltage stabilizer 20, voltage converter 21, converter 22 (type DW-1SG) which is fixed on stick 12, demodulator 23, filter 24, amplifier 25, oscillograph 26. The moment when transient process by applying and removing the control reactions in shape of tensile force starts, is defined by signal from electro pneumatic valve winding.

During testing the object interactions on test reactions, it is necessary to define self inertia of the system's executive element – pneumatic drive. The analysis of the pneumatic element's equation of motion inform, that its dynamical properties can be approximately described by aperiodic link of OT [2,3]:

$$G_{np}(s) = \frac{K_{np}}{T_{np}s + 1},$$

where equivalent time-constant  $T_{np}$  is equal 0,01-0,02s – it depends on initial air-powered cylinder piston position.

They presented the initial coordinate of the object, in which they considered the gain in elastic strains by coordinate  $Y$ , giving the input elementary pitch signal, which takes into consideration the executive element inertia. The operational transmittance is equal

$$\Delta g_y(s) = \frac{1}{s} G_{np}(s) G_{Fg_y}(s) = \frac{K_{np} K_{0Fg_y} (T_{05}s + 1)(T_{06}s + 1)}{s(T_{np}s + 1)(T_{01}s + 1)(T_{02}s + 1)}.$$

The carried out research show, that time-constants in numerator and denominator of OT object have similar value and its dynamical properties are related to properties of proportional link, that's why the transient process by giving the element input pitch is defined mostly by its element properties. For example, by experimental research of time characteristics, they carried out the machining process of the steel 45 part with diameter 5mm and length 200mm.

The machining parameters are:  $a=0,2mm$ ;  $b_l=0,75mm$ ;  $\kappa_r=90^\circ$ ;  $v_c=0,33m/s$ ;  $\tau=0,047s$ , the initial value of tensile force  $F_{x1}=1980N$ .

According to references [2,3], they defined values of factors  $m_x$ ,  $m_y$  are:  $m_x=m_y=0,61 \cdot 10^6 N/m$ . The coefficients of elastic system reaction by coordinate  $X$  are, in that case, defined mostly by machine tool's elastic properties and  $h_{xx}=3 \cdot 10^{-7} m/N$  and by coordinate  $Y$  – by part's elastic properties. Calculations are carrying out according to relations mentioned in [2, 3], for a case when cutting force is applied in the middle point of the part: initial value of part's elastic strains  $g_{y0}=0,11 \cdot 10^{-3} m$ ,  $h_{yy}=1,2 \cdot 10^{-6} m/N$ . The values of factors  $B_0$  and  $B$  are:  $B_0=0,18$ ,  $B=0,17$ .

Taken into consideration, that values of  $B_0$  and  $B$  are close, the object can be considered, in approximation, as proportional link and transient characteristic – defined by executive element inertia. Actually, the experiment result shows, that curve of the transient process has, in that conditions, an exponential course with time constant  $T_{np}$ .

#### 4. SYNTHESIS OF OPTIMAL REGULATOR

The problem of optimal control of TS elastic strains is computed analogous to the one considered in [5,6]. Besides, it is unnecessary to correct the system parameters settings, thanks to quite high stability of the control system parameters. The control system properties are similar to proportional link parameters, according to the results mentioned above. In lots of cases, interferential reaction (in shape of machining allowance change) can be considered as normal stationary random process and the random correlations function of the interferences' coordinate can be approximated by exponential-cosines relation:

$$K(\tau) = D e^{-\alpha\tau} \cos\beta\tau, \quad (7)$$

where  $D$  – dispersion of the random process,  $\alpha$  – index of correlation function decay,  $\beta$  – frequency of correlation function change.

On equal terms with mathematical model of interferences in form of random process with correlation function (7), the random process of “white noise” type can be considered as the interferential model. The model of “white noise” type as the border case of not correlated process is advised to be applied, when there is no credible information of random process characteristics. It should be taken into consideration, that deviation  $\Delta g$  is considered here as regulated coordinate.

Choosing the proper start of  $\Delta g$  mean value computing, that factor can be reduced to zero. Therefore, the interference is going to be considered as random process with mathematical expectation equal to zero.

In the capacity of optimization criterion for TS, which are exposed to interferences in the stationary random process form, it is suitable to take into consideration [5] the minimum of mean value of deviation square  $\Delta g^2$  with restrictions on the driving power  $u^2$  and describe that criterion in following form:

$$J = m^2 \langle \Delta g^2 \rangle + \langle u^2 \rangle, \quad (8)$$

where  $m$  – indefinite Lagrange multiplier.

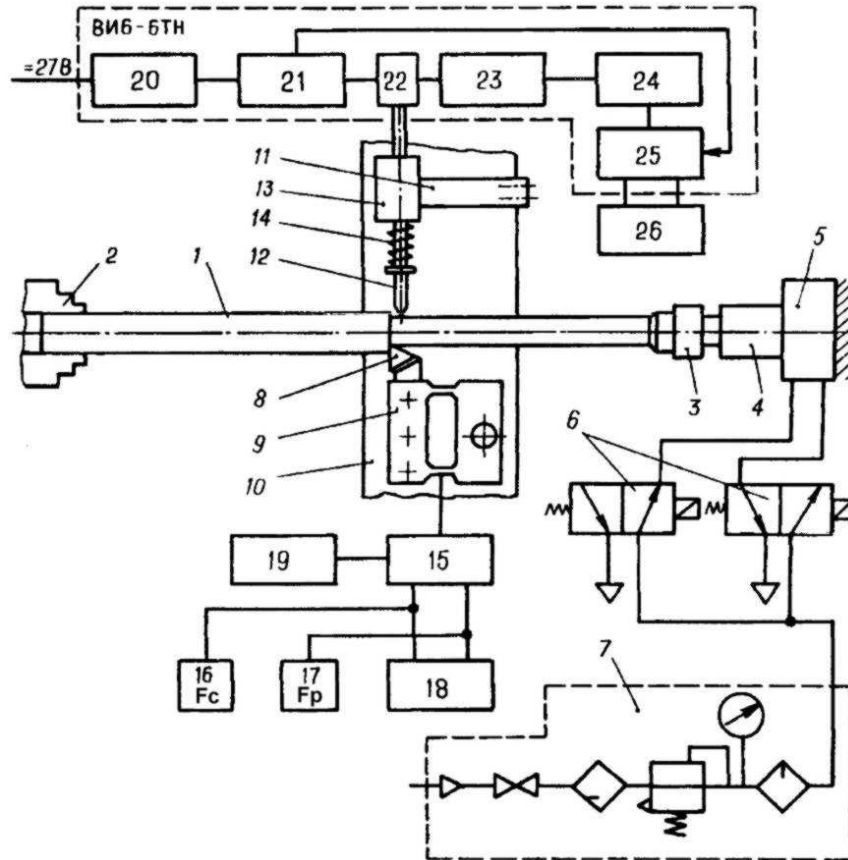


Fig. 1. Block schema of the stand to research the time and frequency characteristics of dynamical system

The operational transmittance is equal (when the generalized object's factor refers to the controller and the proportional link is the dynamical object's model):

$$G_o(s) = \frac{B_o(s)}{A_o(s)} = 1.$$

We synthesize the optimum controller (according to (8) criterion) for interference in the form of random process with exponential-cosines function (7). Standard spectral interference power density:

$$S_\varphi(\omega) = \frac{\alpha^2 + \beta^2 + \omega^2}{(\alpha^2 + \beta^2 + \omega^2)^2 - 4\beta^2\omega^2},$$

Can be defined in the following form (after presenting by Laplace and separating):

$$S_\varphi(s) = S_1(s)S_1(-s),$$

where,

$$S_1(s) = \frac{b_{s0}s + b_{s1}}{a_{s0}s^2 + a_{s1}s + a_{s2}};$$

$$S_1(-s) = \frac{b_{s0}s + b_{s1}}{a_{s0}s^2 - a_{s1}s + a_{s2}}.$$

We can write down the multinomial expression, applying the methodology of synthesis ([4, 5]):

$$G(s)G(-s) = A_o(s)A_o(-s) + m^2 = 1 + m^2.$$

$$G(s) = G(-s) = \sqrt{1 + m^2}.$$

Indirect expression for considered case is:

$$\frac{A_o(-s)}{G(-s)} S_1(s) = \frac{1}{\sqrt{1 + m^2}} \cdot \frac{b_{s0}s + b_{s1}}{a_{s0}s^2 + a_{s1}s + a_{s2}}.$$

Its total part equal zero, because the order of magnitude for denominator's multinomial is higher than numerator's multinomial and the denominator's multinomial of that expression has no solution in left half-plane, therefore:

$$M_0(s) = 0, M_-(s) = 0,$$

$$M_+(s) = \frac{1}{\sqrt{1 + m^2}} \cdot \frac{b_{s0}s + b_{s1}}{a_{s0}s^2 + a_{s1}s + a_{s2}}.$$

There is an expression for auxiliary function:

$$\Phi(s) = \frac{M_+(s)}{G(s)S_1(s)} = \frac{1}{1 + m^2},$$

and the operational transmittance of optimum controller is equal:

$$G_r(s) = A_o(s) - \frac{1}{\Phi(s)} = -m^2.$$

Hence, similarly to the case of interference in the form of random process with exponential-cosines function, the optimum controller is a typical proportional controller, and its proportionality coefficient is described by selected level of limitations on the control reaction.

Similar to previous description, it is easy to obtain expressions for OT of closed system by interference and control, expressions that become proportionality coefficients and after transition to frequency plane – they become expressions for square of amplitude frequency characteristic respecting to control and interfering reaction.

Taken into consideration the obtained correlations, the mean square deviation of initial coordinate  $\Delta g^2$  and mean square value of the control reaction  $\Delta u^2$  are defined.

## 5. CONCLUSION

The results of theoretical and experimental research of object's time characteristics by channel of additional force reactions, confirm the above-mentioned conclusion, that DS's properties are, in approximation, equivalent to proportional link when TS's elastic-deformable state is being controlled. Such simplification is correct only when "low" and "medium" frequencies (dynamical properties of control process and elastic system are not shown) range is being considered. Time-constants of elastic system and cutting process which define limits of the "medium" fre-

quencies range, are between  $0,003s$  and  $0,005s$ . Time-constants of executive element, which are applied during constructing SAC by elastic-deformable state, usually increase the pointed value by an order of magnitude. Hence, the range of important frequencies is defined by the executive element's inertia and is localized more to the left than range of frequencies that are defined by dynamical characteristics of considered object.

In case of interferences in the form of exponential-cosines function, the optimum controller for the model is the typical proportional controller, which proportionality coefficient is defined by selected level of limitations on the control reaction.

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## MODELING PARAMETERS OF THE SURFACE COARSENESS IN AUTOMATED SHAFTS PROCESSING

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*In article was introduced process of roughness parameters modeling by shafts turning. Introduced model comply influence of technological parameters of turning process, turning parameters, stages waste and working period of blade. The comparative analysis of experimental results and computer simulation the mathematical model was introduced.*

Manipulating the products quality at metalwork designing process level requires modeling the process of forming the part quality parameters, e.g. roughness and corrugation. The most important forecasting parameter is quality. This is because defects at final process stage cannot be fixed. Turning process is usually final stage process. High quality of the numerically controlled machines, the high performance of the machining tool and rolling optimal parameters selection guarantees rolling without surplus, burnouts and vibrations.

Control quality parameters is possible both at the stage of designing technology - technological production preparation and as well as at direct control of technological processes and operations. Analysis and optimization of the technology variants is more appropriate at the designing stage. This is because choice of the optimal variant are shortening, new products preparing efforts are decreasing thus simplifying technical resources, directly controlling machining devices. To create such a forecast at the stage of preparing the production and control automatic of parameters quality in metalwork we need to build mathematical model of quality parameters shaping.