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# OPTIMAL DESIGN OF ACTIVE ORTHOSIS WITH REDUNDANT COMPOSITE POLYMER-METAL CONTROLLABLE STIFFNESS ACTUATORS

*Anactive orthosis model with controllable stiffness polymer-metal composite actuators is considered to produce slow (driftless) and accurate limb movements. It can be used for the patients with neuromuscular diseases, such as Parkinson's disease, stroke etc. Optimal stiffness values of actuators, joints and endpointare determined. The relationship between actuators activation, stiffness and control input (voltage) are revealed from the actuator activation dynamics. The optimization approach for redundant actuation system is proposed and the forces developed by actuators are determined. Dynamic modelling using ADAMS program is performed and numerical results are presented.*

***Key words:*** *active orthosis, controllable stiffness, redundant actuation, optimization approach, dynamic modeling*

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**ОПТИМАЛЬНОЕ ПРОЕКТИРОВАНИЕ АКТИВНЫХ ОРТЕЗОВ С РЕГУЛИРУЕМОЙ ЖЕСТКОСТЬЮ КОМПОЗИТНЫХ ПОЛИМЕР-МЕТАЛЛИЧЕСКИХ ПРИВОДОВ**

*Считается, что активная модель ортеза с регулируемой жесткостью полимер-металлических композитных приводов производит медленные и точные движения конечностей. Она может быть использована для пациентов с нервно-мышечными заболеваниями, такими как болезнь Паркинсона, инсульт и т.д. Определены оптимальные значения жесткости приводов, шарниров и крайнего положения. Выявлена взаимосвязь между активацией приводов, жесткостью и входящим сигналом управления (напряжения) от активации динамики привода Предложена. оптимизиция подхода к резервной системе привода и определены силы, выработанные исполнительными механизмами. Выполнено динамическое моделирование с помощью программы ADAMS и представлены численные результаты.*

***Ключевые слова:*** *активный ортез**,** управляема жесткость, резервный привод, улучшенный метод, динамическое моделирование*

## Introduction.

An accurate manipulation of objects in such operations as writing requires slow and accurate movements of the finger and whole limb [1]. Reasons of such inaccuracies can be neuromuscular diseases such as Parkinsons disease, stroke etc. For the rehabilitation purposes we need to develop a fundamentally new type of orthosis which will provide controllable stiffness of joints by the controllable stiffness of joints. And like a human neuromuscular system it is expected that it will have minimal energy consumption.

In controlling actuator stiffness to produce equilibrium limb postures we do not expect a unique solution due to the redundancy of the orthosis actuation system, thus, the same posture can be achieved by multiple combinations of the stiffness parameter values. Therefore, we formulate the problem of actuator stiffness control as an optimization problem that minimizes the strain energy of the limb. We assume that each actuator acts as an ideal spring whose stiffness parameter is set by the electrical command. Then the total strain energy of the system ($E$ ) for the actuators excursions $\Delta s$ is given by

$$E = \frac{1}{2}(\Delta s + \Delta l_0)^T K(\Delta s + \Delta l_0),$$

where the stiffness matrix $K$ is an $m \times m$ diagonal matrix of the entries $k_i\ (i = 1, 2, \dots m)$:

$$K = \begin{pmatrix} k_1 & 0 \\ 0 & k_m \end{pmatrix}.$$

The redundancy leads us to look for the optimal stiffness values $K_{opt}$ that minimize the strain energy function while satisfying the permissible actuator stiffness values, namely:

$$\min_K \left[ E = \frac{1}{2}\left(-R^T\Delta\theta + \Delta l_0\right)^T K\left(-R^T\Delta\theta + \Delta l_0\right)\right],$$

$$-RK(-R^T\Delta\theta + \Delta l_0) = 0,\ k_{min} \le k_1 \le k_{max},$$

where $R$ is the arm moment matrix, $\Delta\theta$ is the angular displacement.

For the next step we should define the joint stiffness and endpoint stiffness [2]. It is an index of mechanical impedance. Considering the upper limb model and assuming a linear relationship between a single actuator activation and the corresponding actuator stiffness, the joint stiffness $K_i(s)$ in the static condition can be expressed as the following function of actuator controlling sums:

$$K_i(c) = k_i \begin{pmatrix} c_s + c_{se} & c_e & c_w + c_{ew} \\ c_e & c_e + c_{se} & c_w \end{pmatrix},$$

where $k_i \left(N \cdot \frac{m}{rad}\right)$ is a gain constant to convert the joint control matrix to joint stiffness.

Assuming that in our case hand force is minimal, the endpoint stiffness $K_{end}(c, \theta)$ can be obtained as follows:

$$K_{end}(c,\theta) = \left(J_{xy}^T(\theta)\right)^{-1} \cdot K_i(c) \cdot J_{xy}^{-1}(\theta),$$

where $J_{xy}^T(\theta) = \frac{\partial(x, y, z)^T}{\partial(\theta_s, \theta_e, \theta_w)}$ is the Jacobian matrix that associates the joint space with the task space in Cartesian coordinates.

Endpoint stiffness also can be expressed by the ratio of applied torque $\mathrm{T}(\mathbf{t})$ to resulting angular velocity $\omega_3$:

$$K_{end}(c,\theta) = \frac{\mathrm{T}(\mathbf{t})}{\omega_3}.$$

The spring-like property of an actuator is approximated by a linear relation between its stiffness $k_i$ and the torque $\tau_i$ that it generates [3]:

$$\tau_i = \rho_i k_i + b_i\ .$$

Here $\rho_i$ is a constant that determines the shape of torque-angle curve, and $b_i$ is passive torque. $\mathrm{T}(\mathbf{t})$ can be expressed as follows:

$$T(t) = \begin{pmatrix} T_1(t) \\ T_2(t) \\ T_3(t) \end{pmatrix} = \begin{pmatrix} \tau_{1a}(t) \\ \tau_{2a}(t) \\ \tau_{3a}(t) \end{pmatrix} + \begin{pmatrix} \tau_{1v}(t) \\ \tau_{2v}(t) \\ \tau_{3v}(t) \end{pmatrix}.$$

The model takes into account the viscous property of the actuator by a lumped viscous torque, which is proportional to the joint stiffness:

$$\begin{pmatrix}\tau_{1v}(t)\\ \tau_{2v}(t)\\ \tau_{3v}(t)\end{pmatrix} = -\delta K(t)\begin{pmatrix}\dot{\theta}_1(t)\\ \dot{\theta}_2(t)\\ \dot{\theta}_3(t)\end{pmatrix},$$

where $\delta$ is a damping coefficient, $\tau_{1v}(t)$, $\tau_{2v}(t)$ and $\tau_{3v}(t)$ are the viscous torques at the shoulder, elbow and wrist joints, $\dot{\theta}_1(t)$, $\dot{\theta}_2(t)$, $\dot{\theta}_3(t)$ are the angular velocities of the shoulder and elbow joints, $\tau_{1a}(t)$, $\tau_{2a}(t)$, $\tau_{3a}(t)$ are the total actuator torque at the shoulder, elbow and wrist joints respectively.

The actuator stiffness is proportional to its activation, such that:

$$K_i(t) = k_i a_i(t) 0 \le a_i \le 1, \quad (1)$$

where $a_i$ is the muscle activation, and $k_i$ is the maximal actuator stiffness achieved at $a_i = 1$ .

Substituting the parameters of polymer-metal composite actuators [4, 5], the equation (1) can be written as follows:

$$K_i(t) = \frac{\left((1-2v)q_i^2 + 4cV_i^2\sigma_i\right)2S_iE}{(1-2v)cu_i^2 + \sigma_i} a_i(t).$$

From this expression we can define the electrical voltage values for each actuator:

$$u_i = \sqrt{\left(\frac{K_i(t)}{a_i(t)\cdot\left((1-2v)q_i^2 + 4cV_i^2\sigma_i\right)2S_iE} - \sigma_i\right)\Big/ (1-2v)c}\,.$$

Where $q$ -electrical charge, $V_i$- the volume of the actuator, $S_i$-the transverse section of the actuator, $v$ - Poisson’s ratio, $E$ -Young’s modulus, $c$ -capacity, $\sigma_i$ -mechanical stress and $u_i$-electrical voltage.

Actuator activation dynamics can be described by a first order equation

$$t_{const} da_i(t) = -a_i(t)dt + u_i(t)dt\,.$$

Here $u_i(t)$ is the electrical control input (voltage) of actuators and $t_{const}$ is the time constant of the actuator activation.

**Redundant control system optimization and determination of actuators forces.**

Various optimization approaches have been proposed to model the optimization principle in redundant actuation systems and to solve this problem by minimizing a cost function [6]. The main difference among these approaches is the structure of cost functions that represent performance criteria based on which the control system optimizes the activation of actuator forces. The most simple and accurate method is using as a cost function the proportion of sum of actuator forces ($F_{m.i}(t)$) and the maximal actuator force ($F_{max.i}$):

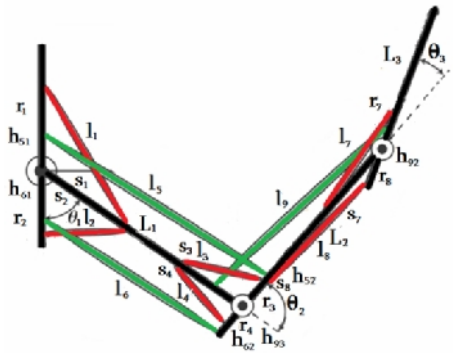
$$\min_K \left[\sum_{i=1}^{n}\left[\frac{F_{m,i}(t)}{F_{max,i}} - A_c\right]^2\right], F_{max,i} \ge F_{m,i} \ge 0$$

where $A_c$ represents the actuator co-contraction, which for slow (drift less) and accurate movements can be chosen from 0.04 to 0.2.

**Kinematics of redundant upper limb actuator system.**

All the actuators can only be linearly deformated and are not curved on the way [7]. The geometric parameters and variables are shown in Figure 1. The lengths of the actuators

($l_i$) are formulated as the distances between the insertion points of the actuators. They can be expressed as follows:



$$l_1 = (r_1^2 + s_1^2 + 2r_1 s_1 \cos\theta_1)^{\frac{1}{2}},$$

$$l_2 = (r_2^2 + s_2^2 - 2r_2 s_2 \cos\theta_1)^{\frac{1}{2}},$$

$$l_3 = (r_3^2 + s_3^2 + 2r_3 s_3 \cos\theta_2)^{\frac{1}{2}},$$

$$l_4 = (r_4^2 + s_4^2 - 2r_4 s_4 \cos\theta_2)^{\frac{1}{2}},$$

Figure 1.Upper limb redundant actuation

$$l_5 = (h_{51}^2 + h_{52}^2 + L_1^2 + 2h_{51}L_1\cos\theta_1 + 2h_{52}L_1\cos\theta_2 + 2h_{51}h_{52}\cos(\theta_1+\theta_2))^{\frac{1}{2}},$$

$$l_6 = (h_{61}^2 + h_{62}^2 + L_1^2 - 2h_{61}L_1\cos\theta_1 - 2h_{62}L_1\cos\theta_2 + 2h_{61}h_{62}\cos(\theta_1+\theta_2))^{\frac{1}{2}},$$

$$l_7 = (r_7^2 + s_7^2 + 2r_7 s_7 \cos\theta_3)^{\frac{1}{2}},$$

$$l_8 = (r_8^2 + s_8^2 - 2r_8 s_8 \cos\theta_3)^{\frac{1}{2}},$$

$$l_9 = (h_{91}^2 + h_{92}^2 + L_2^2 + 2h_{91}L_2\cos\theta_2 + 2h_{92}L_2\cos\theta_3 + 2h_{91}h_{92}\cos(\theta_2+\theta_3))^{\frac{1}{2}},$$

where $l_i (i = 1, 2 \ldots, 9)$ are the lengths of the actuators, and $\theta_i$ $(i = 1, 2, 3)$ are the joint angles. Also, $r_i (i = 1, \ldots, 4, \quad 7, \ldots, 8)$, and $s_i$ $(i = 1, \ldots, 4, \quad 7, \ldots, 8)$ are the distances between the centre of each joint and the insertion point of each mono-articular actuator, and $h_i$ $(i = 51, 52, 61, 62, 91, 92)$ are those of bi-articular actuators.

**Numerical results.**

The admissible joint angles are taken to lie in the range $\theta_1 \in [0^\circ, 90^\circ]$, $\theta_2 \in [-60^\circ, 0^\circ]$, $\theta_3 \in [-30^\circ, 0^\circ]$. For each $\theta_0$ we explore optimal transitions to equilibrium postures in a small square neighborhood centered at 0 and of side length $10^\circ$. We compute the optimal stiffness values for the change in joint angles associated with a transition from the initial posture to each of these postures. Optimal stiffness values $K_{opt}$ depend on $R$ also. Thus for the shoulder joint we will have 9 groups of optimal stiffness values, for the elbow 6 groups and for the wrist 3 groups accordingly. The differences between optimal stiffness values of actuators for each group are not big and for the simplicity of control algorithms and presentation we use their average values:

$$K_{opt\,1} = 124\ N/m, K_{opt\,2} = 118\ N/m, K_{opt\,3} = 113\ N/m, K_{opt\,4} = 100\ N/m, K_{opt\,5} = 600\ N/m, K_{opt\,6} = 544\ N/m, K_{opt\,7} = 100\ N/m, K_{opt\,8} = 120\ N/m, K_{opt\,9} = 550\ N/m.$$

Joints stiffness values also are variable and depend on actuator controlling sums, therefore we use again their average values:

$$K_s = 60\ {}^N/_m,\ K_e = 30\ {}^N/_m,\ K_w = 20\ {}^N/_m,\ K_{end} = 1.3 \cdot 10^{-5}\ Nms.$$

**Dynamic modelling using ADAMS.**

Dynamic modelling is performed with the following parameters of the biomechanical system: $m_{arm} = 2.6\ kg$ , $I^{a}_{XX} = 4.2 \times 10^{-2} kgm^2$, $I^{a}_{YY} = 4.1 \times 10^{-2} kgm^2$, $I^{a}_{ZZ} = 4.4 \times 10^{-4} kgm^2$, $m_{f.arm} = 1.3\ kg$ , $I^{f}_{XX} = 5.1 \times 10^{-2} kgm^2$, $I^{f}_{YY} = 5.1 \times 10^{-2} kgm^2$, $I^{f}_{ZZ} = 8.2 \times 10^{-2} kgm^2$, $m_{hand} = 0.3\ kg$ , $I^{h}_{XX} = 8 \times 10^{-4}$, $I^{h}_{YY} = 7.9 \times 10^{-4}$, $I^{h}_{ZZ} = 1.9 \times 10^{-5}$, and $I_{XY} = 0$ , $I_{ZX} = 0$ , $I_{YZ} = 0$ for all segments. Simulation is started from the equilibrium position.

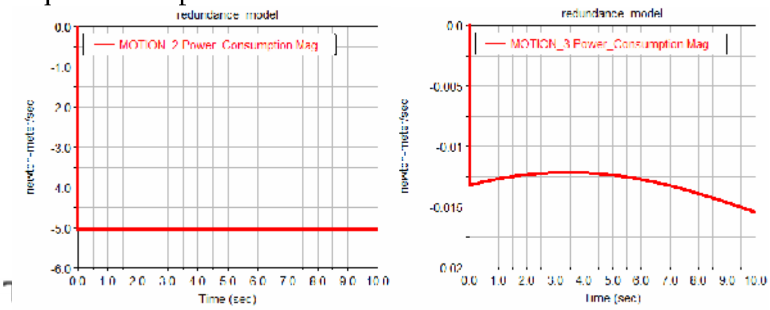



Figure 2. Energy consumption values in shoulder, elbow and wrist joints respectively.

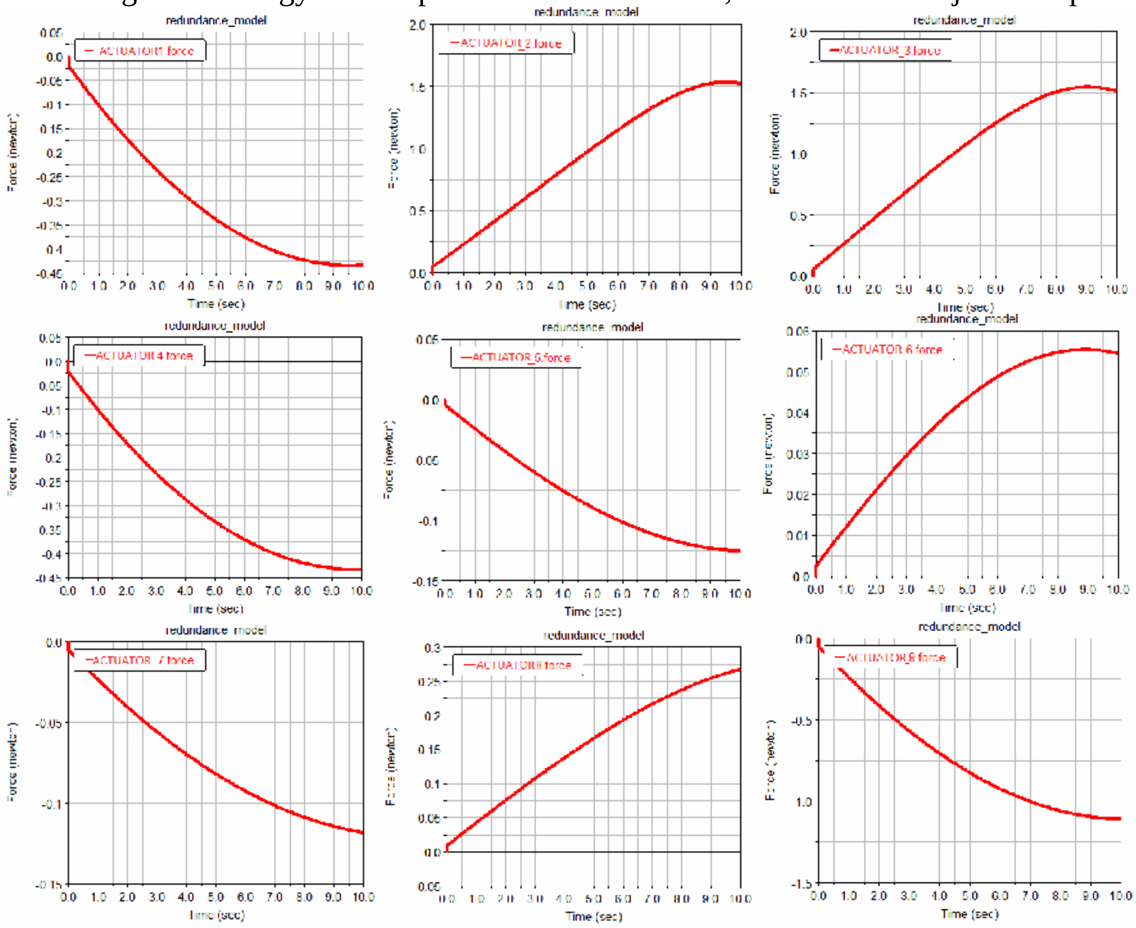



Figure 3.Optimal force values of nine actuators.

**Conclusion.**

Proposed conception of the orthosis actuation system allows providing slow and accurate movements of upper limb and has a great potential for the future clinical applications.

There is a possibility for the finger and lower limb implementations too, which needs just changing the size and the power of actuators. The numerical results show the effectiveness of the proposed redundant actuation system and adventages of controllable stiffness actuators which can a basis for the next generation active orthosis design.

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